THE EFFECT OF CURVATUR OF THE EQUILIBRIUM LINE

ON THE FILM RESISTANCES IN GAS ABSORPTION (II)

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In a gas absorption process the operating line lies above the equilibrium curve. In order to make the operating line straight all the graphs should use weight ratios. Evaluation of the number of transfer units by White's method gave very satisfactory results. In this way much insight can be gained into the operation of gas absorption towers. It is applicable to all tie-line slopes between zero and infinity. There is a critical tie-line slope equal to the operating line slope and of opposite sign. This critical tie-line slope divides the White method into two kinds of steps which are related to the cases in which the gas-film resistance or the liquid-film resistance is most important. At the critical tie-line slope the number of transfer units reaches a maximum and the absorption tower height a maximum.

The water-acetone-air system were used in the exploratory calculation and experiment. The water-acetone-air system has a slightly curved equilibrium curve. An investigation was performed under four different operating conditions using ½-inch Intalox Saddles of 4-inch absorption tower. All experiments were carried out at atmospheric pressure and room temperature.

In this case where the equilibrium curve is almost straight there is a method of analyzing experimental data to determine the approximate tie-line slope. This means that the relative resistances on the liquid-film side and on the gas-film side of the gas-liquid interface can be found. Such information would be quite useful in the design of gas absorption towers.

Experiments were carried out by two identical towers in series-counterflow. Using pure water and acetone was absorbed from air which contained acetone vapor. The gas and liquid compositions at the half way point were determined as well as those at the top and bottom of the system. The number of transfer units were determined by White's method and tried to wide range, from zero to minus infinity, of tie-line slopes. The half way point should on the operating line. The correct tie-line slope would make the half way point identical.

For the conditions tried both calculation and experiment showed reasonable agreement. The variation in the tie-line slope from zero to twenty changed the half way point on the operating line 2 to 8.5% of its length. The true half way point should lie in this ranges, which is shown by experimental results. This gives an idea of the very good accuracy in estimating the true tie-line slope in gas absorption towers.

MASS TRANSFER COEFFICIENT AND

VELOCITY PROFILE

Application of Pohlhausen's Approximate Method for Solving the Boundary Layer Equation to Mass Transfer Problems

Chen Shih-hsueh

I. Introduction

Partial differential equations of the type

$$v\frac{\partial A}{\partial x} = \alpha\frac{\partial^2 A}{\partial y^2} \quad \cdots\cdots (1)$$

Where

v = velocity of fluid

A = transportable quantities such as concentration, $\rho C_p T$, etc.

$x, y,$ = coordinates

α = a physical property of fluid

will meet many engineering situations. For instance, the diffusion of pure gas A through falling liquid film (as in a wet wall column), the equation for this system is

$$V_z\frac{\partial C_A}{\partial z} = D_{AB}\frac{\partial^2 C_A}{\partial s^2} \quad \cdots\cdots (2)$$

With its appropriate boundary conditions, the exact solution of the system was given by Pigford [1] and also by Hatta [2]. According to Hatta the solution is

$$\frac{C_{A1}-C_A}{C_{A1}-C_{Ao}} = \left(\frac{8}{\pi^2}\right)\left[e^{-P_H} + \left(\frac{1}{9}\right)e^{-9P_H} + \left(\frac{1}{25}\right)e^{-25P_H} + \cdots\cdots\right] \quad \cdots\cdots (3)$$

Where

$$P_H = \frac{9}{16}\pi^2\frac{D_{AB}z}{B_f^2 V_s}$$

B_f = thickness of falling film.

The mass transfer rate may be calculated from the concentration distribution directly. But in practical applications, using the mass transfer coefficient $\bar{k}_L$ is more convenient. The $\bar{k}_L$ may be derived from the concentration distribution, but because C_A as in eq. (3) contains n terms, it is not easy to make use of the exact solutions.

Many empirical equations [3] were given for predicting the $\bar{k}_L$ values, but very few workers had paid attention to the theoretical analysis. The Pohlhausen method [4] for solving the boundary layer equation is introduced for mass transfer from solid shape to liquid system, and the result is checked with experiments and also compared with the empirical equations. It is found that the Pohlhausen method is very useful for predicting the functional dependence of $\bar{k}_L$ or mass transfer j_D factor for the given system.

II. Theoretical Analysis

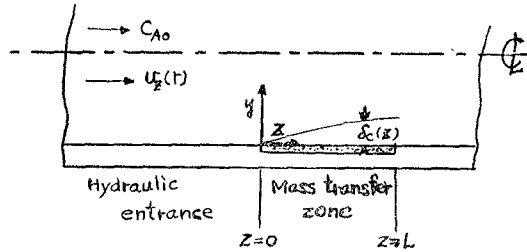For the system shown below:

Fig·1

in mass transfer zone, the wall is coated with salicylic acid (which is only sparingly soluble in water). The whole wall is connected smoothly so that we can assume no roughness is introduced. For this system the following assumptions will be reasonable:

(1) Assumptions

    i) Surface concentration of salicylic acid in water at mass transfer zone is its saturated concentration.

    ii) Mass flux of salicylic acid is very small, (this is true under these experimental conditions) that the flux neither changes the velocity plofile nor changes the physical properties of fluid.

    iii) Neglect the roughness introduced by dissolving out of the coated salicylic acid. (In this experiment, the quantity which dissolved are kept under 100 mg.)

    iv) Steady state.

(2) Fundamental equations.

    i) From momentum balance

$$O = \frac{1}{\rho}\frac{dp}{dz} + \nu \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

    ii) From shell mass balance of species A in mass transfer zone

$$v_z\frac{\partial C_A}{\partial z} = D_{AB}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right) + \frac{\partial^2 C_A}{\partial z^2}\right] \quad \cdots\cdots\cdots\cdots\cdots\cdots (5)$$

the term $D_{AB}\dfrac{\partial^2 C_A}{\partial z^2}$ may be neglected in comparison with the $v_z\dfrac{\partial C_A}{\partial z}$ term so that eq. (5) may be reduced to

$$v_z\frac{\partial C_A}{\partial z} = D_{AB}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (6)$$

(3) Solution for laminar flow.

    Eq. (4) may be integrated to get

$$v_z = v_{z\,max}\left[1-\left(\frac{r}{R}\right)^2\right] = 2\bar{v_z}\left[1-\left(\frac{r}{R}\right)^2\right] \cdots\cdots\cdots\cdots\cdots (7)$$

Substitute eq. (7) into eq. (6) get

$$2\bar{v_z}\left[1-\left(\frac{r}{R}\right)^2\right]\frac{\partial C_A}{\partial z} = D_{AB}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right) \quad \cdots\cdots\cdots\cdots (8)$$

Now assume similar concentration distribution holes in the mass transfer zone that lead to

$$C_A^* = \frac{C_{As}-C_A}{C_{As}-C_{Ao}} = C_A^*(\eta) = a\eta + b\eta^2 + c\eta^3 + d\eta^4 \quad \cdots\cdots\cdots\cdots (9)$$

$$\eta = \frac{R-r}{\delta_c(z)} \dots\dots\dots (10)$$

the coefficients a, b, c, d in eq. (9) may be determined by boundary conditions:

$$\text{B.C.1;} \quad \eta=0 \quad C_A^*=0, \quad \frac{\partial^2 C_A^*}{\partial \eta^2}=0 \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \dots\dots\dots (11)$$

$$\text{B.C.2;} \quad \eta=1 \quad \frac{\partial C_A^*}{\partial \eta}=0, \quad \frac{\partial^2 C_A^*}{\partial \eta^2}=0, \quad C_A^*=1$$

the eq. (9) will be found to be

$$C_A^*=2\eta - 2\eta^3 + \eta^4 \dots\dots\dots (12)$$

substitute eq. (9), (10), and (12) into (8) and integrate to get

$$\left(\frac{2\bar{v_z}\delta_c}{R^2 D_{AB}}\right)\left(\frac{d\delta_c}{dz}\right)\int_0^1 \eta^2(2R-\delta_c\eta)(R-\delta_c\eta)(2-6\eta^2+4\eta^3)d\eta$$

$$= \int_0^1 (R-\delta_c\eta)(12\eta-12\eta^2)d\eta + \delta_c\int_0^1(2-6\eta^2+4\eta^3)d\eta \dots\dots (13)$$

or $$\left(\frac{2\bar{v_z}}{R^2 D_{AB}}\right)\left(\frac{d\delta_c}{dz}\right)\left(\frac{4}{15}R^2\delta_c^2 - \frac{3}{14}R\delta_c^3 + \frac{3}{70}\delta_c^4\right)=2R \dots\dots\dots (14)$$

integrate eq. (14) again to get

$$\left(\frac{2\bar{v_z}}{R^2 D_{AB}}\right)\int_0^{\delta_c}\left(\frac{4}{15}R^2\delta_c^2 - \frac{3}{14}R\delta_c^3 + \frac{3}{70}\delta_c^4\right)d\delta_c = \int_0^L 2R\,dz \dots\dots (15)$$

or $$\frac{8}{45}\left(\frac{\delta_c}{D}\right)^3 - \frac{3}{14}\left(\frac{\delta_c}{D}\right)^4 + \frac{12}{175}\left(\frac{\delta_c}{D}\right)^5 = \left(\frac{D_{AB}}{\bar{v_z}D}\right)\left(\frac{L}{D}\right) \dots\dots\dots (16)$$

or this case, L is not too large, then $\frac{\delta_c}{D}$ will be very small, so the 4th. and 5th. power terms may be neglected.

Then

$$\frac{\delta_c}{D}=1.776\left[\left(\frac{D_{AB}}{\nu}\right)\left(\frac{\nu}{D\bar{v_z}}\right)\right]^{\frac{1}{3}}\left[\frac{L}{D}\right]^{\frac{1}{3}} \dots\dots\dots (17)$$

From definitions

$$\left.\begin{matrix} \bar{\delta_c} = \dfrac{\int_0^L \delta_c\,dz}{L} \\[4ex] k_L = \dfrac{D_{AB}}{\delta_c} \\[4ex] \bar{k_L} = \dfrac{D_{AB}}{\bar{\delta_c}} \end{matrix}\right\} \dots\dots\dots (18)$$

get

$$\bar{k_L}=0.757\left(\frac{D_{AB}}{D}\right)(Re\cdot Sc)^{\frac{1}{3}}\left(\frac{L}{D}\right)^{-\frac{1}{3}} \dots\dots\dots (19)$$

or $$j_D = \frac{\bar{k_L}}{\bar{v_z}}Sc^{\frac{2}{3}}=0.757(Re)^{-\frac{2}{3}}\left(\frac{L}{D}\right)^{-\frac{1}{3}} \dots\dots\dots (20)$$

(4) Solution for turbulent flow

It is known that in fully developed turbulent flow, the velocity profiles by logarithmic law are:

$$\left.\begin{matrix} v_z^+ = y^+ & y^+ < 5 \\ v_z^+ = -3.05 + 5\ln y^+, & 5 < y^+ < 30 \\ v_z^+ = 5.5 + 2.5\ln y^+, & y^+ > 30 \end{matrix}\right\} \dots\dots\dots (21)$$

If the condition $\delta_{c\,(max)} \leqslant y|_{y^+=5}$ holds, then the eq. (6) will be

$$v_z\left(\frac{\partial C_A}{\partial z}\right)=y^+v^*\frac{\partial C_A}{\partial z}=(R-r)\frac{v^{*2}}{r}\frac{\partial C_A}{\partial z}=D_{AB}\left(\frac{\partial^2 C_A}{\partial r^2}+\frac{1}{r}\frac{\partial C_A}{\partial r}\right)\cdots\cdots\cdots\cdots(22)$$

Use the same assumption as in the Laminar Flow case

$$\left.\begin{aligned}C_A*&=\frac{C_{As}-C_A}{C_{As}-C_{Ao}}=2\eta-2\eta^3+\eta^4\\\eta&=\frac{R-r}{\delta_c(z)}\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(23)$$

Substitute in eq. (22) and integrate to get

$$\left(\frac{v^{*2}}{\nu}\right)\left(\frac{d\delta_c}{dz}\right)\int_0^1\eta^2(R-\delta_c\eta)(2-6\eta^2+4\eta^3)d\eta$$

$$=\left(\frac{D_{AB}}{\delta_c{}^2}\right)\left[\int_0^1(R-\delta_c\eta)(12\eta-12\eta^2)d\eta+\delta_c\int_0^1(2-6\eta^2+4\eta^3)d\eta\right]\cdots(24)$$

or $\quad\left(\dfrac{v^{*2}}{\nu}\right)\left(\dfrac{1}{15}D-\dfrac{1}{14}\delta_c\right)\left(\dfrac{d\delta_c}{dz}\right)=\left(\dfrac{D_{AB}}{\delta_c{}^2}\right)D\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(25)$

Integrate again and get

$$\frac{1}{45}\left(\frac{\delta_c}{D}\right)^3-\frac{1}{56}\left(\frac{\delta_c}{D}\right)^4=\left(\frac{\overline{v_z}}{v^*}\right)^2\left(\frac{D_{AB}}{\nu}\right)\left(\frac{\nu}{D\overline{v_z}}\right)^2\left(\frac{L}{D}\right)\cdots\cdots\cdots(26)$$

Neglect $(\delta_c/D)^4$ term then

$$\frac{\delta_c}{D}=2.67\left(\frac{\overline{v_z}}{v^*}\right)^{\frac{2}{3}}(Sc)^{-\frac{1}{3}}(Re)^{-\frac{2}{3}}\left(\frac{L}{D}\right)^{\frac{1}{3}}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(27)$$

From an empirical turbulent velocity profile

$$\left(\frac{\overline{v_z}}{v^*}\right)^2=\frac{2}{f}=\frac{1}{0.023\ Re^{-0.2}}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(28)$$

and from eq. (27), finally get

$$\frac{\delta_c}{D}=9.4(Sc)^{-\frac{1}{3}}(Re)^{-0.6}\left(\frac{L}{D}\right)^{\frac{1}{3}}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(29)$$

$$\overline{k_L}=\frac{D_{AB}}{\delta_c}=0.1063\left(\frac{D_{AB}}{D}\right)(Sc)^{\frac{1}{3}}(Re)^{0.6}\left(\frac{L}{D}\right)^{-\frac{1}{3}}\cdots\cdots\cdots\cdots(30)$$

or $\quad j_D=0.1063(Re)^{-0.4}\left(\dfrac{L}{D}\right)^{-\frac{1}{3}}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(31)$
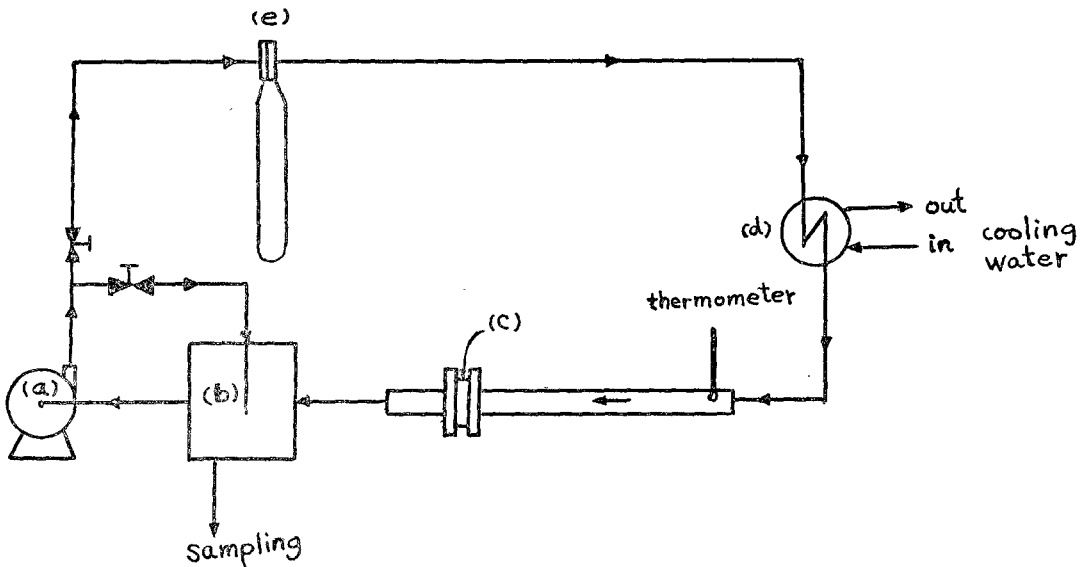
III. Experiment

Fig-2

The schema of the experiment is shown in fig-2. There (a) is a cycling pump, (b) is a mixing tank, (c) is mass transfer section, (d) is a heat exchanger, and (e) is a manometer. The detail of the mass transfer zone is shown in fig-3.
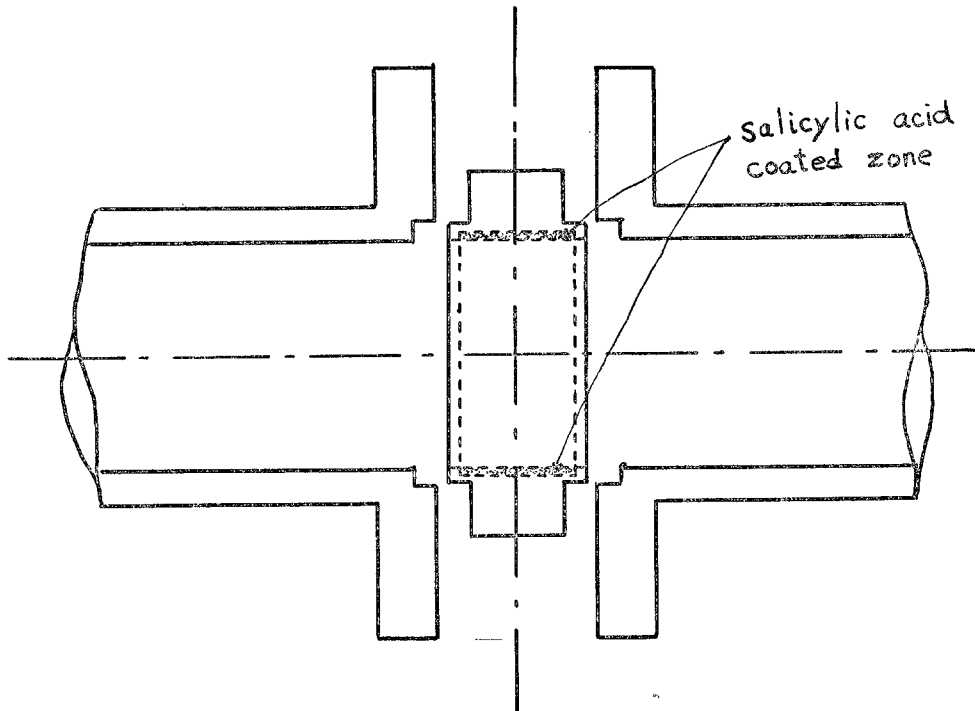

salicylic acid
coated zone

fig-3 part (c) of fig-2

Salicylic acid is coated in the groove of sample holders and made smooth with respect to the pipe inside diameter. A small quantity of sample water is withdrawn at 5 to 10 mim. intervals, and the concentration of salicylic acid in water is determined by a spectrophotometer. (Color developer is 2 drops of $FeCl_3$ soln.) In the experiment, max. acid concentration is about 35 mg. per liter. In this range, absorbance at $525m\mu$ shows very good linearity with acid concentration.

The mass transfer coeficient $\overline{k_L}$ is calculated as follow:

　　Let

　　　　W_A = quantity of the acid dissolved in unit time

　　　　C_{As} = saturated acid concentration

　　　　C_A = bulk concentration of the acid

　　　　A　= transfer area of the system

　　　　H　= total liquid hold-up

From mass balance

$$H\frac{d\overline{C}_A}{dt} = W_A = \overline{k}_L A(C_{As} - \overline{C}_A)$$

C_{As} is about 2200 mg/1, and $\overline{C}_A$ varies from O mg/1 to 35 mg/1, so that

$\triangle C_A = C_{As} - \overline{C}_A$ we may regard as a constant.

$$\therefore \quad \overline{C}_A = \overline{k}_L \left(\frac{A}{H}\right) \triangle C_A t + k$$

Therefore in a plot of $\overline{C}_A$ vs t, the slope must be $\overline{k}_L\left(\frac{A}{H}\right)\triangle C_A$.

By measuring the solpe we can calculate $\overline{k}_L$.

IV. Result

1. Effect of mass transfer area on mass transfer coefficient.

In the system, A is proportional to L. According to this analysis $\overline{k}_L$ is proportional to $L^{-\frac{1}{3}}$ or $A^{-\frac{1}{3}}$. Three different sizes of sample holders are made for checking the effect of L on $\overline{k}_L$. The following are results at Re=1350.
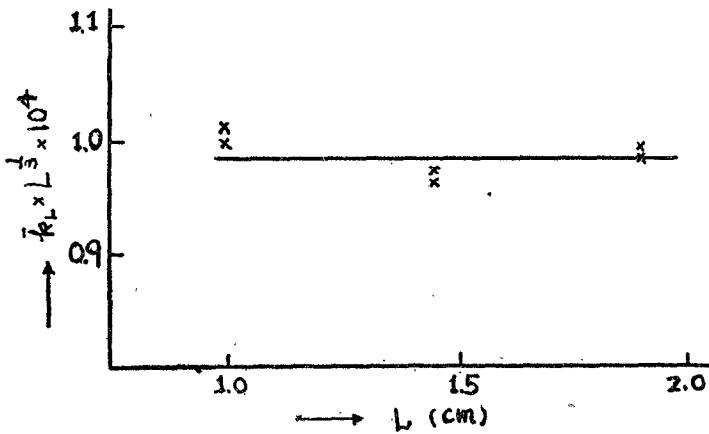

Fig-5 $\overline{k}_L$ vs L

The proportionality of $k_L \propto L^{-\frac{1}{3}}$ is fairly good in the experiment ranges.

2. Effect of Re on mass transfer coefficient.

Experiments are run for different flow rates with the L=1.4 cm sample holders. The results are plotted on the following figure.
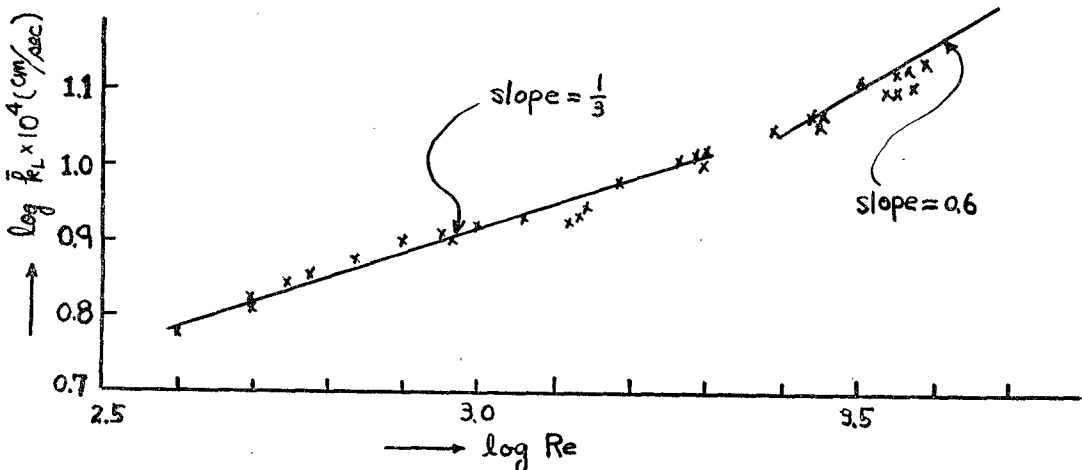

Fig-5 $\overline{k}_L$ vs Re

V. Conclusion

From modified Pohlhausen's method, the mass transfer j_D factors are:

$$j_D = 0.757(Re)^{-\frac{2}{3}}\left(\frac{L}{D}\right)^{-\frac{1}{3}} \cdots\cdots\cdots\cdots\cdots\text{for laminar region}$$

$$j_D = 0.1063(Re)^{-04}\left(\frac{L}{D}\right)^{-\frac{1}{3}}\cdots\cdots\cdots\cdots\text{for turbulent region}$$

According to Lington & Sherwood's empirical equations:

$$j_D = 1.61(Re)^{-\frac{2}{3}}\left(\frac{L}{D}\right)^{-\frac{1}{3}} \cdots\cdots\cdots\cdots\cdots\text{for laminar region}$$

$$j_D = 0.232\,(Re)^{-0.4}\left(\frac{L}{D}\right)^{-\frac{1}{3}}\cdots\cdots\cdots\cdots\text{for turbulent region}$$

In the two results, only the dimensionless coefficients differ by about two times. The coefficient for these expeiments are not known because no reliable Sc number of salicylic acid in water may be found. By using the Sc number of benzoic acid which possesses the same physical and chemical properties, these coefficients are very close to the results of the empirical formula by Lington & Sherwood. Therefore Pohlhausen's method is successful for predicting the functional dependence in mass transfer operations but the coefficients will not be accurate. This may be accounted for by the singularity introduced at the inlet portion.

Nomenclature

A = mass transfer area

C_A = concentration of species A, C_{Ao} = initial conc., C_{A1} = interface conc.,

C_{As} = saturated conc., $\overline{C_A}$ = local bulk conc., C_A^* = dimentionless conc.

C_p = heat capacity

D = diameter

D_{AB} = diffusivity of A in B

f = friction factor

H = liquid hold-up

j_D = mass transfer j factor

k_L, $\overline{k_L}$ = local and mean mass transfer coefficients

L = distance related to mass transfer area

p = pressure

R = radius

T = temperature

t = time

v_x, v_z, v_r = velocity components, v^* = friction velocity, v^+ = dimentionless velocity

x, y, z, r, s, = coordinates, $y+ = \dfrac{yv^*\rho}{\mu}$ = dimensionless coordinate

W_A = rate of A dissolved

Re = Reynolds number

Sc = Schmidt number

ρ = density of fluid

ν = kinematic viscosity

δ_c = concentration boundary layer thickness

μ = viscosity

References:

(1) Pigford; Ph. D. thesis, Univ. of Illinois (1941)

(2) 八田四郎次; 工化 37, 601 (1934)

(3) W. H. Lington; T.K. Sherwood, Mass transfer from solid shapes to water in stream & turbulent flow, Chem. Eng. Progress, Vol. 46, No. 5 p 258-64 (1950)

(4) Schlichting, Boundary Layer Theory, p. 130, p. 229

物質傳送係數與速度分布之關係
Pohlhausen 之境界層方程式解法之應用

陳 世 學

從圓管管壁向管內流動中之流體之物質傳送，如傳送速度緩慢時可得方程式如下

$$v_z \frac{\partial C_A}{\partial z} = D_{AB} \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial C_A}{\partial r}\right)$$

應用 Pohlhausen 之境界層方程式之近似解法可解得上式。其結果與實驗結果，以及其他實驗式比較，除所得係數稍小之外方程式內各項皆與實驗式或實驗結果吻合。是故 Pohlhausen 之近似解法爲一簡單而有效之方法。

MASS TRANSFER COEFFICIENT AND
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Slow rate of mass transfer from circular pipe wall to steady flow fluids can be described by following equation:

$$vz \frac{\partial C_A}{\partial z} = D_{AB} \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial C_A}{\partial r}\right)$$

The equation may be solved by "the Pohlhausen's approximate Method for Solving Boundary Layer Equation". The result shows very good prediction of functional dependence in correlation, but the coefficient is smaller than the empirical formula.